

# Homework I

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## 1 3D transformations

**Exercise 1.** Prove Rodrigues' formula: for every unit-length vector  $r$  and angle  $\alpha$ , the matrix of equation 1 (see *rotations* hand-out) is a rotation matrix that corresponds to a rotation of  $\alpha$  radians around the vector  $r$ . (hint: first consider the case  $r = (0, 0, 1)^t$ ; for the general case consider two other unit-length vectors  $u$  and  $v$  that jointly with  $r$  define an orthogonal matrix  $Q = [uvr]$ , and study the matrix  $QRQ^t$ ).

**Exercise 2.** Implement Rodrigues' formula. Validate the routine by computing the rotation matrix  $R$  that encodes a rotation by  $\theta = 30$  degree around the vector  $n = (1, 1, 1)^t$ .

**Exercise 3.** Implement the inverse of Rodrigues' formula. Validate the routine by verifying that, given the  $R$  from ex.2, the output is again  $r = n/\|n\|$  and  $\theta = 30$ .

**Exercise 4.** Rodrigues' formula defines a function

$$\{\omega : 0 < |\omega| \leq 1\} \rightarrow \text{SO}(3),$$

with  $s = (|\omega|^2)^{1/2}$ ,  $c = (1 - |\omega|^2)^{1/2}$ , and  $r = \omega/s$ , which is clearly 1-1 and continuous. Show that the function is well defined and continuous at  $\omega = 0$ .

**Exercise 5.** What rotations matrices correspond to  $|\omega| = 1$ ? What angle of rotation these matrices correspond to?

## 2 Triangulation

**Exercise 6.** Implement the triangulation code in `StereoTriangulator.java` and test it on the five data sets provided. Save the reconstructed 3D objects as VRML files (FILE>SAVE>WRL). To show the reconstructed geometry, choose different viewpoints and save the corresponding images.

## 3 Error Analysis

**Exercise 7.** In this exercise we want to verify that the reconstruction error of the triangulation procedure is function of the baseline. Start your Java application and load "bun000.msh" (FILE>LOAD>WRL 3D MODEL). Then, select 5 different baselines (e.g. you can start from the default one and then use the "half baseline" button to decrease it) and save each stereo pair images by using the (FILE>SAVE>STEREODATA) command. Then load the 5 stereo pair images you have generated and triangulate each of them. After each triangulation, save the 3D coordinates by using the (FILE>SAVE>3D COORDINATES) command. A bun000.3dp file is generated each time. We provide a file (bun000real.3dp) with the real 3D object coordinates of the bunny. Plot the reconstruction

error  $E$  as function of the baseline. The reconstruction error may be expressed as follows:

$$E = \sum_{n=1}^N (X_n^e - X_n^r)^2 \quad (1)$$

where  $X_n^r$  is the 3D coordinate vector of the  $n^{th}$  reconstructed point and  $X_n^e$  is the 3D coordinate vector of the  $n^{th}$  real point.  $N$  is the total number of 3D points. Comment the results.

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